The Natural Rate of Punishment January 2012

By Jan Anders Sorensen University of Southern Denmark

Part of this chapter is an edited Conference paper: CLEA 2012

University of Toronto

Sometimes we do not realize how fragile our civilization really is

F.A. von Hayek

Abstract:

This paper investigates a model where people in an isolated geographic area are able to allocate working time between two sectors in the economy. They can choose between a productive sector, where the real wages are determined solely by the capital stock in the economy. Or they can choose a "non-legal" sector, where the real wages are determined by "income- transfers" of other members of society. As a consequence of decreasing return to scale, this "non-legal" sector would quickly run out of "low-fruit" and easy money to acquire, and one would, therefore, expect a steady state and simple overall equilibrium.

However, as people move to the non-legal sector and crime rises, the anticipated return on capital becomes smaller. This would hurt the accumulation of capital and actually lower the real wages, making the crime sector look even more appealing. Even if one included the powerful forces from the Inada conditions, whereby the return of capital would become higher as the capital stock turned lower, the negative forces from crime could be so overwhelming that it could set-off a negative and totally destructive spiral.

Everything is critically dependent on the central planner's ability to maintain law and order, or more precisely, to obtain a "natural rate of punishment." If the actual punishment is greater than the natural rate of punishment, a society would accumulate more and more capital, which in turn would make the productive sector more competitive and lower the appeal of the crime sector.

As a society becomes richer, the natural rate of punishment would fall, thereby making room for more "humanism". However, if the central planner fails to generate a sustained level of punishment, then the natural rate of punishment would increase.

The main conclusion from this model is the need to be very careful when attempting to export legal systems and moral values from richer countries to poorer ones.

Introduction

On the surface, there only seem to be small differences between the economic “free choice” approach to crime and the positivistic “criminals are sick” approach. Many criminologists would argue that mans’ behavior is determined by biological or sociological factors beyond their control, which implies the effect of deterrence is rather small. (Gottfredson 1990, Sherman 1992, Raine 1993). Economists, on the other hand, argue that criminals in general are highly adaptive, and we, at least at the aggregate level, should foresee that an expected level (or degree) of punishment would highly influence criminal behavior (Becker 1968, Becker 1976, Levitt 2006, Polinsky 2006). At first glance, it seems most differences between the positive style criminologist and the economist are simply a dispute about the deterrence effect, and therefore, an empirical question about the elasticity of prices related to the supply of crime.

But the differences are greater than this. Digging deeper, implicit or explicit, many economists or classical criminologists in the style of Hammurabi (king of Babylon born, 1792 BC.) or Bentham, are often concerned with the long-term dynamics of crime. Punishment is primarily a matter of principle, a way of equalizing things between the offender and the victim(s) (Miller 2005). It is misleading to think that punishment is simply a pedagogical tool, and therefore one to be evaluated on its efficacy of teaching people how to behave. For example, consider the classical criminologist Marxism view[1] that punishment of the thief is merely an intervention to benefit the ruling class. To see the Marxian argument clearly, suppose a central planner wants to maximize:

(1) $$U(N) = \sum_1^N u_i$$

N is the total population in that society. Normally, we believe that utility is a decreasing function of income (Y), which could be formally stated, as:

(2) $$\frac{\partial u_i}{\partial y} > 0, \frac{\partial^2 u_i}{\partial^2 y} < 0, \lim_{y\to\infty} u'(y) \to 0\,, \lim_{y\to 0} u'(c) \to \infty$$

[1]See Chambliss, W. (1973)."Elites and the Creation of Criminal Law" in *Sociological Readings in the Conflict Perspective* Chambliss, W. (ed.) Reading, Mass.: Addison-Wesley. (pp 430–444).

Therefore, optimization of utility by the rule in (4), implies that $\frac{\partial u_i}{\partial y} = \ldots\ldots \frac{\partial u_j}{\partial y}$, meaning that the marginal utility of one extra dollar should be equalized as the optimum result.

Hence, from this utility perspective, it is not at all clear why punishing the poor thief could generate higher overall welfare. At first sight, it seems the overall welfare rises if a poor man steals from the rich. One could, of course, argue like Becker(1968), that "stealing" is a costly form of transfer of wealth between people, and results in lower total welfare.. But it surely depends on the benefits at the margin, and this cannot generally be a clear-cut argument for why societies have historically been very harsh in their treatment of thieves.

However, most people do not feel comfortable with this sort of academic reasoning. There is, without a doubt, a "feeling" that stealing is not just wrong, but is a general threat to society at large, and will create chaos if not disciplined. Surprisingly, very few academics, have tried to investigate this "feeling", or more precisely, a "dynamic" view of crime and its effect on economic growth and capital accumulation. (Detotto 2010) tries to estimate the effect of crime on economic growth in Italian data, and finds it to be significant. (Mongrain 2011) studies inter-jurisdictional competition and its effect on local capital accumulation. But to the best of my knowledge, there have been very few attempts to put the question of crime into a more general macroeconomic framework.

A model of the natural rate of punishment

For centuries, the classical capital accumulation model by Solow has led to hundreds of articles and is now standard in every textbook regarding economic growth. However, there have been few attempts to take a closer look at the discount rate, which simply stated is a homogenous and an exogenous variable[2]. But assume, in general, this is not the case. For instance, according to standard utility theory, poor people have a very high utility on

[2] I have not found that view in other places; not even in the *Handbook of Social Choice and Welfare,* Chap.11, "Utilitarianism and the Theory of Justice" by Blackorby, Bossert, and Donaldson. (2001)

consumption on the margin. This implies that low-income nations should have a higher discount factor than rich countries do. On the other hand, as people become richer, the utility of consumption becomes smaller, which means it is easier to secure private property, and it follows that crime and punishment could become an endogenous variable[3]. So, by the logic of the Inada conditions, as capital stock moves lower, the powerful forces of these conditions would ensure the return of capital to a higher level, which in turn would then stabilize the economy. However, as people get poorer, the discount rate could go higher, and in some cases overwhelm the forces of a higher return of capital. Therefore, it is possible that multiple equilibria exist.

Which solution a society will end up with is critically dependent on the forces/protection of private property implemented by a central planner. Hence, it seems that legal institutions' protection of private property is crucial, and that the failure to implement the correct degree, or a "natural level," of punishment could be fatal for any economy. It is the relative punishment that matters, not the absolute! If this theory is correct, it could explain why poor countries must have very severe punishment for violation of property rights when compared to rich countries. It implicitly follows that any "export" of "moral values" from rich countries to poor countries could be counter-productive.

The model

<u>Allocating time between productive work and "involuntary transfer of wealth" work.</u>

We investigate an economy that consists of n number of self-employed workers, whose only interest is to maximize income. There is no specific preference for any type of human action.

[3] Surprisingly little attention has been giving to the question of stability of the discount rate. In an older paper by Tapan Mitra (International Economic Review, Feb. 1979 vol.20, nr.1), the mathematical problems, letting the discount rate, varying over time, are analyzed. The conclusion is, overall, that there exists no simple solution, if the discount rate should be allowed to vary arbitrarily.

And there is no substitution between leisure and work. Let the unit 1 be the total working time; then the problem is to allocate time between two sorts of work, $l \; and \; \theta$. Hence the time restraint is:

$$(3) \qquad \mathrm{l} + \theta = 1 \qquad \text{where } \mathrm{l} \wedge \theta \in \mathbb{R}_{+} \leq 1$$

l could be seen as productive work, which at the aggregate level is a factor in production, Y, and therefore, overall tends to means a higher standard of living. θ is time involved in involuntary redistribution or transfer of <u>expropriated</u> income from person to person, which at the aggregate level, is not able to raise the standard of living, but for the individual seems like a substitute for an higher income. Normally, θ could be regarded as crime, but one should note, that θ is not always, in legal terms, a crime, because it depends on the legislation from men in power[4]. However, we will assume that θ is to be defined as a crime, meaning that the central planner does not like that action. Therefore, the criminal activity comes with a probability of getting caught and punished. We denote τ as the expected cost of committing a crime, which in principle, does not exclude any individual discount factor, ϑ, meaning that $\tau(\theta) = \int_a^b e^{-\vartheta t} c(\theta)\, dt$. If c is the cost of committing a crime, then τ is the diluted value of that cost. Some criminologists and also some economists seem to believe that the criminals' discount factor is very important, and should be included in the analysis[5]. Indeed, if $\lim_{\vartheta \to \infty} \tau(\theta) -> 0$, then deterrence is not possible. This seems like a very extremist view,

[4]Natural-law theory, therefore, distinguishes between "criminality" (which derives from human nature) and "illegality" (which originates with the interests of those in power). Lawyers sometimes express the two concepts with the phrases *malum in se* and *malum prohibitum* respectively. They regard a "*crime malum in se*" as inherently criminal; whereas a "*crime malum prohibitum*" (the argument goes) counts as criminal only because the law has decreed it so.

[5]Gottfredson and Hirschi: *A General Theory of Crime* (1990) or J. Mccrary . J *Dynamic Perspectives on Crime* (2009), nber working paper, UC Berkeley

and nothing, in the dynamic sense, would be gained by the analysis because the agent is optimized at each point in time.[6]

Let, w_1 be the market wages, and let $w_2$be the return of crime, then the representative agent's problem is to maximize:

(4)
$$\max \mathrm{I} = \mathrm{W}_1\,\mathrm{l} + \mathrm{W}_2(\theta) * \theta - \tau(\theta)$$
$$s.t.\ l + \theta = 1$$

Hence, the Lagrange function is:

(5)
$$\ell = \mathrm{W}_1 \mathrm{l} + \mathrm{W}_2(\theta) * \theta - \tau(\theta) - \varkappa(1 - \mathrm{l} - \theta)$$

We will assume that $\tau'(\theta)>0$, which means the total amount of punishment is an increasing function of the crime activity.

This problem is, of course, closely related to the theory of crime and thinkers like Bentham (1789), Becceria (1763), and Becker(1967). However, instead of thinking in terms of utility, we are thinking in terms of income, and regarding criminal activity as a way of creating income. Therefore, it should be reasonable to determine W_2 as function of crime activity in itself, meaning that any criminal activity faces decreasing return to scale. Therefore, $\lim_{\theta \to 0,}\mathrm{w}_{2\to\infty}$, and $\lim_{\theta\to \mathrm{T},}\mathrm{w}_{2\to 0}$. The logic is simply that there will always be some very profitable crime projects in every society, but as more people move into those areas, the profit opportunities will vanish. Because of the law of diminishing return, you will never "win" the fight against crime, but it should indeed be possible to minimize it.

---

[6]In another paper I have explained why an infinite discount rate is unrealistic and incompatible with the idea of " free will"

The first order condition is:

(6) $$\frac{\partial I}{\partial \theta} = -W_1 + W_2'(\theta) - \tau'(\theta) = 0$$

Or

(7) $$W_1 + \tau'(\theta) = W_2'(\theta)$$

That simply says if someone wishes to maximize income, they should commit a crime until the marginal benefit is equal to the marginal cost. (Opportunities cost + marginal cost of doing crime)[7]

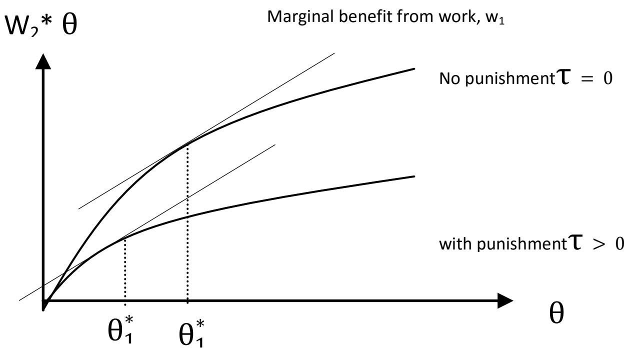


Figure 1: Higher punishment on any given action θ would lower that action)[8]

[7]Hence, we are following the first rule of Bentham (1931) as "The evil of the punishment must be made to exceed the advantage of the offense."

[8]I have elsewhere argued that the idea that crime and punishment is non-elastic is more or less an extremist view, and should include very special ideas of individual discounting. People with an infinite discount factor, I believe, are the economic equivalent of being "sick" and irrational.

Income from crime

As noted before, W_2 is determined by the properties $\lim_{\theta\to 0,}\ w_{2\to\infty}$, and $\lim_{\theta\to T,}\ w_{2\to 0}$. This is exactly in line with most criminology literature[9], where crime activity faces a diminishing return. As an example, we could specify the function W_2 as:

(8) $$W_2 = \varphi/\theta^a$$

Where$a \in [0;1]$ and φ is some constant. If $\theta \to 1$, then $W_2 \to \varphi$, meaning that the benefit of committing a crime will eventually "dry out" and move to some exogenous factor φ, (for example, expropriate from foreign countries) as more time is allocated to this sort of activity,

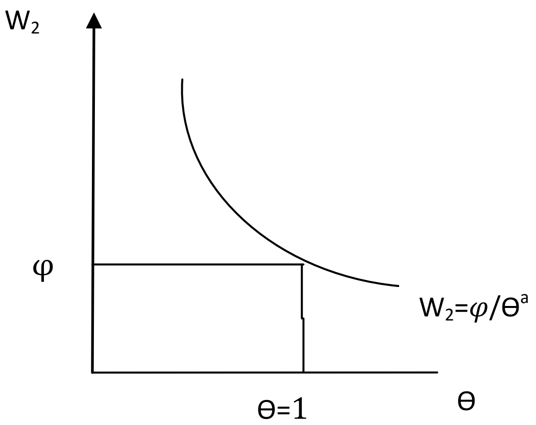


Figure 2: As more time is allocated to "crime" the benefit pr. time unit will fall until some exogenous minimum

Then income generated in this sector of the economy is $w_2\Theta = (\varphi/\ \Theta^a)\ \Theta$ or:

(9) $$W_2\Theta = \varphi\theta^\alpha,$$

[9]As an example, Marceau and Mongrain Competition in law enforcement and capital allocation, *Journal of Urban Economics* 69 (2011) 136-147

where $\alpha = 1 - a$, therefore $\alpha \in]0;1[$

Suppose that we are able to describe the punishment, imposed by the central planner for doing θ, as a linear function. Then the total cost of doing θ is $\tau\theta$. Therefore, the problem from (7) becomes:

(10)

$$\mathrm{MaxI} = \mathrm{w}(\mathrm{T} - \theta) + \varphi\theta^{\alpha} - \tau\theta$$

The first order condition for this problem is:

(11)

$$\frac{\partial \mathrm{I}}{\partial \theta} = -\mathrm{w} + \alpha\varphi\theta^{\alpha-1} - \tau = 0$$

The solution with respect to θ is:

(12)

$$\theta = \left(\frac{\alpha\varphi}{\tau+\mathrm{w}}\right)^{1/(1-\alpha)}$$

With the restriction that $\theta \in]0;1[$

We should note that $\lim_{\tau+\mathrm{w}\to\infty}\left(\frac{\alpha\varphi}{\tau+w}\right)^{\frac{1}{1-\alpha}} \to 0,$ meaning that as wages or punishment moves higher, the allocation of "involuntary distribution of welfare" moves to zero. So, one should expect that rich societies with high market wages and a severe degree of punishment also would have a very low crime rate. On the other hand, one should expect that economies with an inefficient central planner and a very low market wags would be a very dangerous place if one wishes to accumulate capital[10].

The economy

We investigate an economy which has the following production function:

(13) $$y = F(k, l)$$

We assume the function F follows the Inada conditions: this means the production function f, $\mathbb{R}_+ \to \mathbb{R}_+$ should satisfy $f_k(0) = 0$, $f_k^{'}(0) = \infty$, $f_k'(\infty) = 0$, $f_l(0) = 0$, $f_l^{'}(0) = \infty$, and $f_l'(\infty) = 0$. Further, we assume y (•) is homogenous with degree 1 for all $k, l \in \mathbb{R}_+$

Barelli and Pessoa (2003) show the Inada condition means the production function must be asymptotically Cobb-Douglas, which means we could proceed with a production function:

(14) $$F(k, l) = k^{\beta} l^{1-\beta}$$

If r is the price of one unit of capital and w the price of one unit of labor, where $l \epsilon [0; T]$, this implies that: the first order condition must satisfy:

[10] I believe that Singapore and Somalia could be seen as two extreme cases. Violating private property in Singapore could return severe degree of punishment, where the same sort of violation in Somalia, in some cases, could turn you into a hero.

(15) $$\frac{\partial F}{\partial l} = 0 \quad \Rightarrow F_l'(k, l) = w$$

(16) $$\frac{\partial F}{\partial k} = 0 \quad \Rightarrow F_k'(k, l) = r$$

Or, in the case of a Cobb-Douglas function:

(17) $$\mathrm{Mrts} = \frac{\beta l}{(1-\beta)k} = \frac{r}{w}$$

Which in turn gives us a demand for labor as:

(18) $$l^* = \frac{rk((1-\beta)}{w\beta}$$

And of course,

$$\frac{\partial l^d}{\partial w} < 0, \frac{\partial^2 l^d}{\partial^2 w} < 0, \lim_{w\to\infty} l^d \to 0, \text{ and } \lim_{w\to 0} l^d \to T.$$

Therefore, for any given capital stock and real interest, the demand for labor should be a strictly decreasing function of wages.

The capital stock could be uniquely determined by:

(19) $$k^* = \frac{w\beta l}{r(1-\beta)}$$

$$\frac{\partial k^*}{\partial r} < 0, \frac{\partial^2 k^*}{\partial^2 r} < 0, \lim_{r\to\infty} k^* \to 0, \text{ and} \lim_{r\to 0} k^* \to \infty.$$

This implies, that the capital stock should be a decreasing function of r

It can be proved with (17), which in this case is very important. In other words, that w could be determined as a constant fraction of F/L=y, meaning wages are a constant fraction of capital pr. labor. So, wages in any given time, t, is given by;

(20) $$w_t = a * y_t$$

Where

(21) $$y=F/L=k^{\beta}$$

The evolution of capital is determined by:

(22) $$K_{t+1} = K_t + I_t - \delta K_t$$

Where δ is the deteriorated ratio of capital.

Furthermore, we suggest that investment is a function of Y, and the saving ratio of that income;

(23) $$I_t = sy_t$$

Solving for I_t gives us:

(24) $$K_{t+1} = K_t + sk_t^{\beta}\text{-}\delta K_t$$

So, capital evolved according to (25), which is standard.

Savings and the risk premium

In the standard model it is assumed the saving rate, *s*, must be an exogenous variable. But this assumption critically depends on the assumption of institution as a given fact. One of the main findings from the literature of crime is that we cannot be sure that such an institutional setup — which allows savings — is a given situation simply because it is a dominant strategy to steal from people who accumulate capital. In one respect, institutions of private property solve a huge prisoner problem. Any savings, and therefore the return of psychical or human capital, depends in a crucial manner on the expectation of being "expropriated" by taxes or worse by the government, or "expropriated" by other individuals (crime) .

Let us assume the savings ratio could be described as:

(25) $$s = s_o + s(\theta)$$

S_0 is the savings ratio, which is determined by some exogenous factors. $s(\theta)$is the savings ratio, directly related as a function to the crime rate and therefore the probability being expropriated. So one could argue that $s(\theta)$ is the risk premium regarding crime.

It is reasonable to suggest that:

$\frac{\partial s^*}{\partial \theta} > 0, \frac{\partial^2 k^*}{\partial^2 r} > 0, \lim_{\theta \to 1} s^* \to 0$, and $\lim_{\theta \to 0} s^* \to s_0$.

The risk premium will, therefore, accelerate as the probability of expropriation rises.

At this point, it is important to once again to make the point that this kind of risk premium could be very large in societies: not only those with weak governments, but also those with strong governments whose goal it is to expropriate income from savings. Suppose, for example, that $\rho(\theta)$ is the probability of being robbed and losing everything. Suppose an investor choose I_0 as an investment. He would then get rI_0 in the next period. r is interest rate. In total his investment in period 1, would then be $I_0 + rI_0$. The expected value $E(I_0 + rI_0)$ would then be defined by $\rho(\theta)$ and by the law of arbitrage;

(26) $$(I_0 + rI_0) = \rho(\theta) * 0 + (1 - \rho(\theta))(I_0 + r_o I_0 + r_\theta I_0)$$

Where r_θ is the risk premium regarding expropriation and r_o is the interest rate from the standard model.. Isolating r_θ from above will give:

(27) $$r_\theta = \frac{(1+r_0)\rho(\theta)}{1-\rho(\theta)}$$

If the risk of being robbed and losing anything from the initial investment is estimated to be 0, 3 and $r_0 = 0{,}15$ then the risk premium is above 49% in the period under consideration. It is easy to see that $\lim_{\rho(\theta)\to 0} r_\theta \to 0 \; and \; \lim_{\rho(\theta)\to 1} r_\theta \to \infty$. Then, if investors think there is a significant chance that they will lose their initial investment, the risk premium could make the arbitrage large.

The saving function

When the risk premium is higher, the saving ratio is of course lower, so the saving ratio must be somewhat determined by the probability of being robbed. As an example in this qualitative analysis, let us assume we should be able to describe the saving ratio as:

(28) $$S = S_o(1 - \theta^{\pi})$$

π is a parameter > 1, catching how sensitive the savings really are when more people allocate time into crime. So, if one were to anticipate a new regime, which would expropriate everything, then $\theta \to 1$, the risk premium moves to infinity, and $s \to 0$, then any capitalistic economy will deteriorate very quickly[11].

Putting it together and solving the model

Using (29) and (25) gives us:

(29) $$K_{t+1} = (1 - \delta)K_t + S_0[1 - \theta^{\pi}]k_t^{\beta}$$

Please note that when $\theta = 0$, the steady state solution becomes $\vec{K} = \left(\frac{\delta}{s_o}\right)^{\frac{1}{\beta-1}}$ as we, of course, should expect from the textbook solution. Also note that if $\theta = 1$, then $\vec{K} = 0$, which also should be of no surprise. The main problem lies in what we should expect when 0<$\theta < 1$. To see that, setting (13) to (30) produces

(30) $$K_{t+1} = (1 - \delta)K_t + S_0\left[1 - \left(\frac{\alpha\varphi}{\tau + ak_t^{\beta}}\right)^{\pi/(1-\alpha)}\right]k_t^{\beta}.$$

[11] I believe this is somewhat in line with Hayek's statement that "we are not always aware how fragile our civilization really is," citation from "*Commanding Heights*" by Daniel Yergin and Joseph Stanislaw.

This different equation says that capital stock will grow by the net effect from savings, but somehow will be dampened by the effect from criminal activity. The crucial problem is how big this dampening effect will be. It depends critically on the relative prices of being an expropriator and a productive worker; this is determined by the ratio $\frac{\varphi}{\tau+w(k)}$.

Steady state.

We define a steady state solution as $K_{t+1} - K_t = 0$, which gives us:

(31) $$-\delta K_t + S_0[1-\theta^{\pi}]k_t^{\beta} = 0,$$

or

(32) $$S_0 k_t^{\beta} - \delta K_t - S_0 k_t^{\beta}\left(\frac{\alpha\varphi}{\tau+\sigma k_t^{\beta}}\right)^{\frac{\pi}{1-\alpha}} = 0.$$

Unfortunately, we are not able to isolate k in (32) by standard algebraic methods, but we should be able to clearly see that $\lim_{\varphi\to 0}\left(\frac{\alpha\varphi}{\tau+k_t^{\beta}}\right)^{\frac{\pi}{1-\alpha}} \to 0$. If $\lim_{\tau\to\infty}\left(\frac{\alpha\varphi}{\tau+\sigma k_t^{\beta}}\right)^{\frac{\pi}{1-\alpha}} \to 0$, and if $\lim_{k_t^{\beta}\to\infty}\left(\frac{\alpha\varphi}{\tau+\sigma k_t^{\beta}}\right)^{\frac{\pi}{1-\alpha}} \to 0$, then in all cases the solution collapses to the steady state standard solution $\vec{K} = \left(\frac{\delta}{s_o}\right)^{\frac{1}{\beta-1}}$. That could be considered good news because a central planner might indeed somehow control τ, and therefore, also the amount of criminal activity and implicitly, the accumulation of capital.

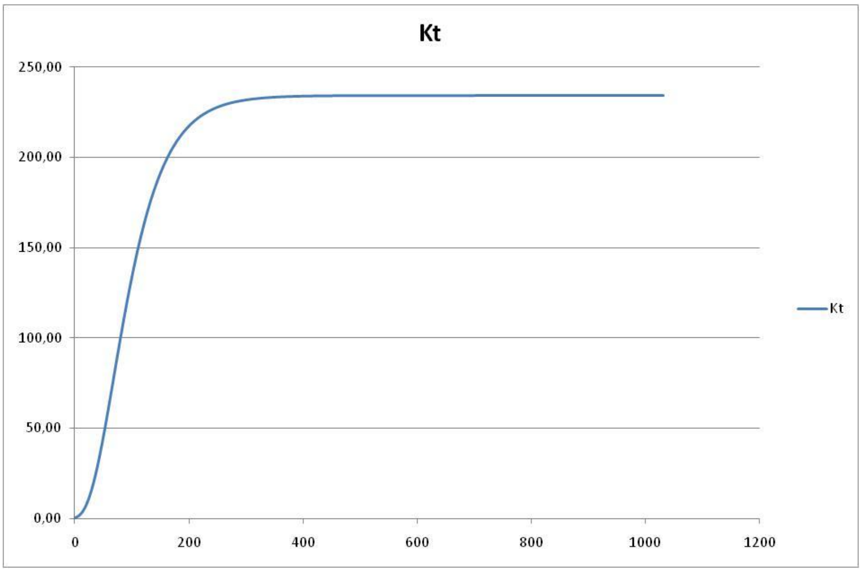
However, things can get rather complicated fast. The reason is $\alpha < 1$, which means that $\frac{1}{1-\alpha} > 1$, which suggests crime grows exponentially when $\alpha\varphi > \tau + \sigma k_t^{\beta}$, but when $\alpha\varphi < \tau + \sigma k_t^{\beta}$, it deteriorates very quickly. The force of θ $(crime)$ under some circumstances is powerful enough to overwhelm the extremely powerful force of diminishing return to scale.

Let us numerically investigate some cases.

Numerical analysis

Case 1: (the standard case)

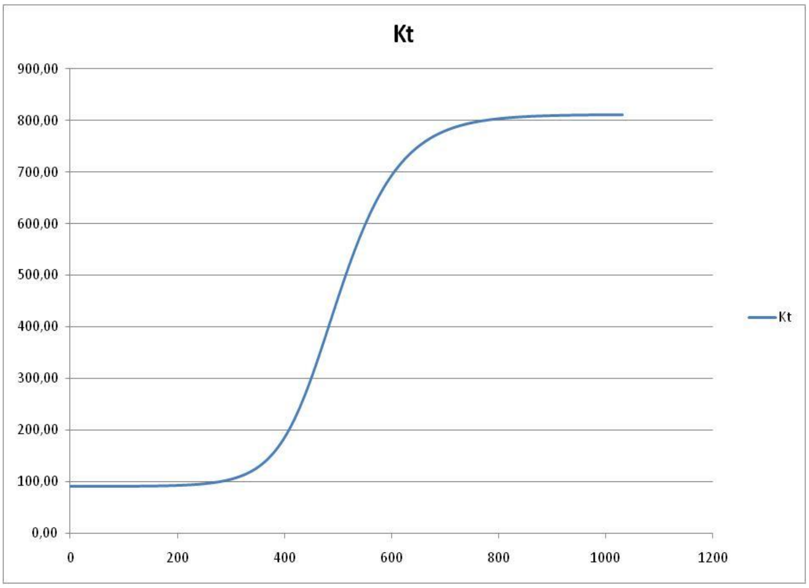
φ (base rate in sector 2)	10
α	0,3
τ (punishment)	40
β	0,8
a (wage ratio)	0,7
so	0,3
δ	0,1



Because $< \tau + \sigma k_t^{\beta}$, it should be of no surprise that this parametric value will converge to the textbook steady state. There are some incentives to move at sector 1, but only for a very small fraction of agents. In this case, θ=0,007 and ss=234. If $\tau \to \infty$, a steady state will be 243. Hence, this scenario confirms the classical view that “crime” is simply a lump sum cost for society.

Case 2 (Seemingly stable for a long period of time, then suddenly explodes)

φ (baserate)	80
α	0,5
τ	38,845
β	0,8
a (wage ratio)	0,7
so	0,4
δ	0,1

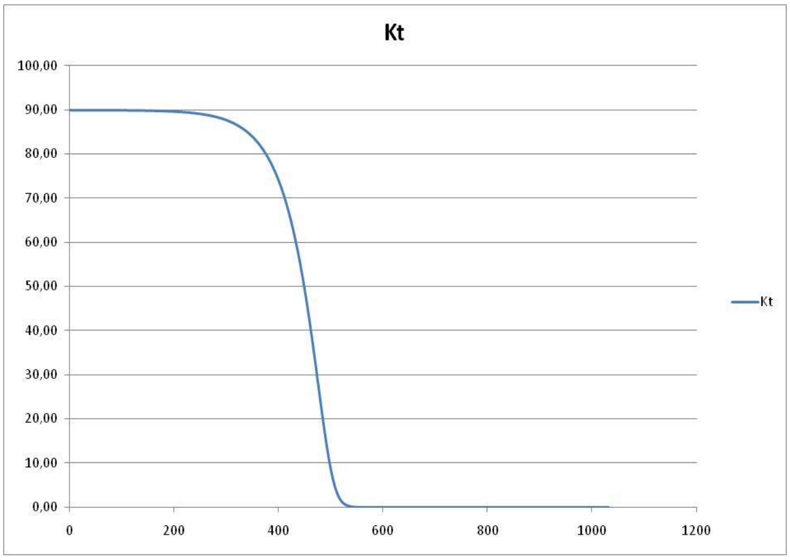


Ss=811,2

In this case, the exogenous rate φ is relatively high compared to the competition from wages in the capitalistic sector. There seems to be a steady state, but the steady state is unstable. In this case it would in the end converges to the “rich” state.

Case 3 (Seemingly stable for a long period of time, then suddenly deteriorates)

φ (base rate in sector 2)	80
α	0,5
τ	38,84
β	0,8
a (wage ratio)	0,7
so	0,4
δ	0,1



SS=0

This case illustrates the unstable situation. Parametric value is the same as in case 2. The only difference is a slight reduction in punishment. Then, the economy collapses and we are able to suggest the existence of multiple equilibria and the implicit existence of "A NATURAL RATE OF PUNISHMENT". Should punishment be below the natural rate, the economy will deteriorate, because it becoming less and less competitive being in the accumulative sector. This in turn would deteriorate savings, which, for some parametric value, would overwhelmed the strongly forces from higher and higher return on capital.

If punishment is higher than the natural rate, capital would be accumulated, then raise the wages and creating more incentives for people moving out of criminal activity and then make a natural defense against crime. This would further stimulate the saving ratio. In such case the economy simply would converge to some classic steady state.

Evaluating the parametric value in steady state.

Proving the existence of multiple equilibria depends on the differential parametric values in the steady state. We know that:

$\frac{dk_{t+1}}{dk_t} > 1$, then the steady state is unstable. But when $\frac{dk_{t+1}}{dk_t} \leq 1,$ the steady state is stable. We therefore want to differentiate the function:

(33)

$$K_{t+1} = (1-\delta)K_t + S_0\left[1-\left(\frac{\alpha\varphi}{\tau+ak_t^{\beta}}\right)^{\pi/(1-\alpha)}\right]k_t^{\beta}$$

(34)

$$\frac{dk_{t+1}}{dk_t} =$$

$$1-\delta+\left[\frac{S_0k_t^{\beta}\beta}{k_t}\right]-\left[\frac{S_0k_t^{\beta}\beta(\frac{\alpha\varphi}{\tau+\sigma k_t^{\beta}})^{\frac{\pi}{1-\alpha}}}{k_t}\right]+\left[\frac{\left(S_0k_t^{\beta}\right)^2(\frac{\alpha\varphi}{\tau+\sigma k_t^{\beta}})^{\frac{\pi}{1-\alpha}}}{(1-\alpha)(\tau+\sigma k_t^{\beta})k_t}\pi\beta\right]$$

Evaluating the slope in those steady states is because 1+2+3 gives the following: (0,98, 0,99, 1,02). Because case 3 has a differentiated value of 1,02 in steady state, the steady state must be unstable. We should note that the general solution, where

$\lim_{\left(\frac{\alpha\varphi}{\tau+k_t^{\beta}}\right)^{\frac{1}{1-\alpha}}\to 0}\frac{dk_{t+1}}{dk_t} = 1-\delta+\left[\frac{S_0k_t^{\beta}\beta}{k_t}\right]<1$, is the textbook solution, implying only one single stable steady state.

This produces the conclusion shown in the following diagram:

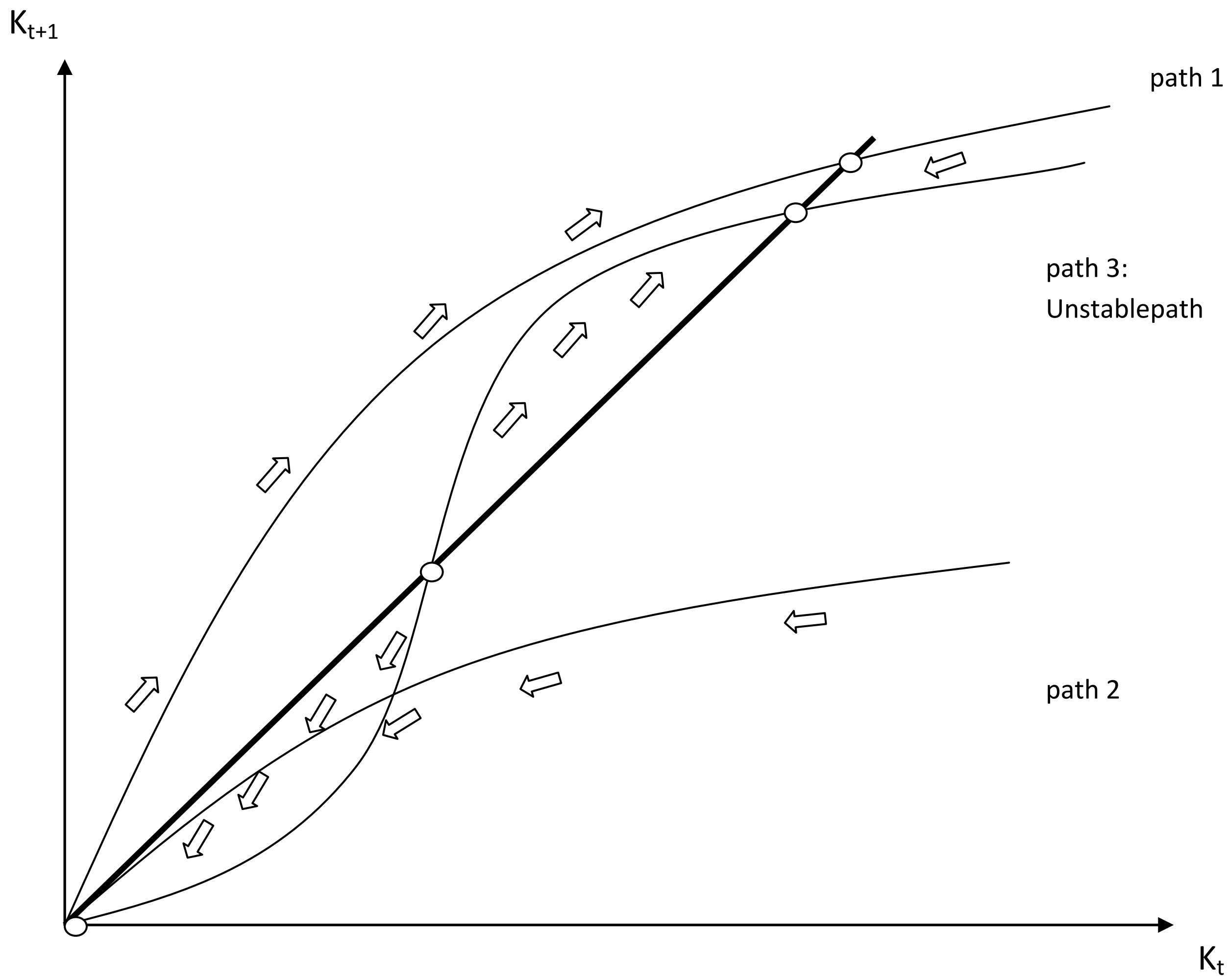


Figure 3: The phase-diagram, showing three different paths to equilibrium.

In path 1, a strong "capitalist-friendly" planner keeps a high level of punishment. Therefore, more capital will be accumulated, people become richer, and the incentives to steal from the capital base — with regard to punishment — become smaller. Because of a rising marginal return for any criminal activity, there will always be some crime, and the steady state solution would be a little smaller than that one known from the standard model. In path 2, if a central planner seeks to destroy the capital base, or allows people to steal, the economy would of eventually end up with zero capital. In path 3, there exists a capitalist-friendly central planner, but the parametric values mean the path is unstable. If such a planner tries to be friendly toward people committing crime, or alternatively, allows too much "welfare" by stealing from the capital base, income could fall, and thus create an even greater incentive to expropriate wealth because the discount rate turns higher.

The movement of the natural rate of punishment

The central planner controls the parameter τ;, therefore, in principle he also controls which steady state the economy will reach in the longer term. For simplicity, suppose the central planner has a discount factor of 1, then the income maximization problem becomes:

(35)
$$\mathrm{Max}\ \mathrm{I} = \sum_0^T \sigma k_t^\beta (1-\theta_t) + \ \varphi\theta_t^\alpha - \tau\theta_t$$

s.t.
$$K_{t+1} = (1-\delta)K_t + S_0[1-\theta_t]k_t^\beta$$

The decentralized solution was

(36)
$$\left(\frac{\alpha\varphi}{\tau+\sigma k_t^\beta}\right)^{1/(1-\alpha)}$$

which then means the central planner must try to control:

$$J_s(x) = \max \left[\sigma k_t^{\beta} - \sigma k_t^{\beta}\left(\frac{\alpha\varphi}{\tau+\sigma k_t^{\beta}}\right)^{\frac{\pi}{1-\alpha}} + \varphi\left(\frac{\alpha\varphi}{\tau+\sigma k_t^{\beta}}\right)^{\frac{\pi\alpha}{1-\alpha}} - \tau\left(\frac{\alpha\varphi}{\tau+\sigma k_t^{\beta}}\right)^{\frac{\pi}{1-\alpha}} + J_{s+1}\left[K_t - \delta K_t + S_0 - \left(\frac{\alpha\varphi}{\tau+\sigma k_t^{\beta}}\right)^{\pi/(1-\alpha)} k_t^{\beta}\right]\right], \quad s=0,1,,,,, T\text{-}1$$

or;

(37)

$$J_T = \max_{k\in K} \sigma k_T^{\beta} - \sigma k_T^{\beta}\left(\frac{\alpha\varphi}{\tau+\sigma k_T^{\beta}}\right)^{\frac{\pi}{1-\alpha}} + \varphi\left(\frac{\alpha\varphi}{\tau+\sigma k_T^{\beta}}\right)^{\frac{\pi\alpha}{1-\alpha}} - \tau\left(\frac{\alpha\varphi}{\tau+\sigma k_T^{\beta}}\right)^{\frac{\pi}{1-\alpha}}$$

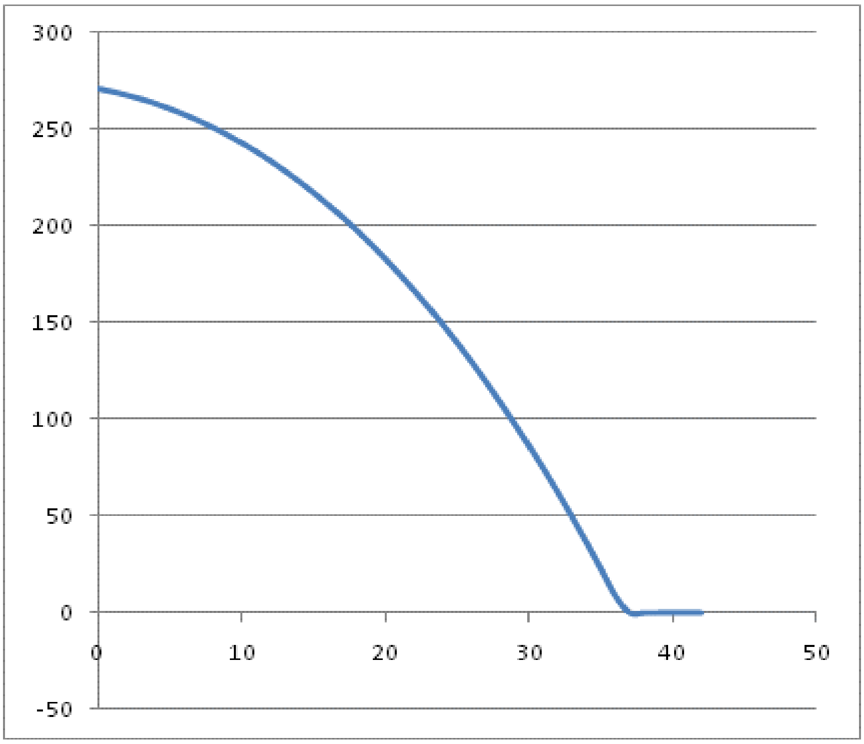
This is the fundamental equation of dynamic programming. Moving backwards from T, it is easy to see that if the central planner has an very low horizon, he would set $\tau = 0$, implying that he would expropriate all the goods from the "capitalist". However, when the time horizon becomes larger, the cost of not having a high punishment becomes ever greater.

Unfortunately, there is no simple solution to the dynamic problem, but we can simulate a solution as:

The optimal path for parametric values:

α	0,5
ψ	40
β	0,8
a wage ratio	0,7
s_o	0,4
δ	0,1

N=30, K-start=81



Hence, it is evident that a central planner must set a very high level of punishment when the economy is weak. But as the economy grows, incomes increase, which in turn make crime a relatively less attractive business. Therefore, the NATURAL RATE OF PUNISHMENT FALLS as the economy grows, which in turn could lead a central planner to impose more "humanized" types of punishment, or alternatively, allow some degree of expropriation. Therefore, the model predicts, as an important second order conclusion, that we should see a very tough central planner in low-income economies, and a more "human" central planner in high-income economies. Hence, it could be problematic to try to implement a "civilized" system of punishment in a very poor economy because that could reduce the punishment below its natural rate.

Of course, some may ask, what will happen if the time horizon is not definite? Will that make any difference? Sometimes it would. But in this case it will not. Things just become slightly more difficult, because we then must incorporate a time-discount factor into the problem. Using the maximum principle to find the solution of the Hamiltonian:

(38)

$$H = \sum_0^\infty b^t \left[\sigma k_t^\beta - \sigma k_t^\beta \left(\frac{\alpha\varphi}{\tau+\sigma k_t^\beta}\right)^{\frac{1}{1-\alpha}} + \varphi \left(\frac{\alpha\varphi}{\tau+\sigma k_t^\beta}\right)^{\frac{\alpha}{1-\alpha}} - \tau \left(\frac{\alpha\varphi}{\tau+\sigma k_t^\beta}\right)^{\frac{1}{1-\alpha}} + \lambda \left[K_t - \delta K_t + S_0 - \left(\frac{\alpha\varphi}{\tau+\sigma k_t^\beta}\right)^{1/(1-\alpha)} k_t^\beta \right] \right],$$

,

In this case, the curve would only become smoother as time moves forward.

Conclusion

In this paper, the overall hypothesis is that people in countries with low income do have a higher discount factor than people in high-income countries. Poor people, therefore, have a higher incentive to allocate time to the business of expropriating welfare from others. As the capital stock grows larger, the incentives to allocate time to legal and productive work will become larger. However, if a central planner fails to implement necessary institutions and a high enough expected penalty for crime, this could hurt capital accumulation. The effect could be so devastating that it could overwhelm the Inada conditions, and thereby create a downward economic spiral. Therefore, strong institutions are critical for a stable growth path to exist. Humanism can become a luxury good a very poor country simply cannot afford. Clearly, the exportation of institutions from rich to poor countries could become a very dangerous strategy.

References

Bentham J. (1789). "The Principle of morals and legislation." http://www.econlib.org/library/Bentham/bnthPML1.html

Beccaria (1764). "Dei Deliti e Delle Pene (on crimes and Punishment). "http://www.constitution.org/cb/crim_pun.htm

Barelli P., de Abreu Pessoa S. (2003), Inada conditions imply that production function must be asymptotically Cobb-Douglas, Economic Letters 81(3) pp.361-363.

Becker.G (1965)_A Theory of the Allocation of Time, The Economic Journal, Vol. 75, No. 299, pp. 493-517.

Becker G. (1968)*,* "Crime and Punishment: An Economic Approach“ *The Journal of Political Economy*, 76(2),pp.169-217

Becker G. (1976). *The Economic Approach to Human Behavior*. University of Chicago Press.

De Soto, Hernando (2000). *The Mystery of Capital: Why Capitalism Triumphs in the West and Fails Everywhere Else*. New York: Basic Books.

Detotto, C. Otranto E (2010). *Does Crime Affect Economic Growth?* Kyklos 63(3): pp.330-345

Gottfredson M. R. , Travis H. (1990). A General Theory of crime, Standford university Press.

Gronau, R. (1977): “Leisure, Home Production, and Work: The Theory of the Allocation of Time Revisited," *Journal of Political Economy*, 85(6) pp.1099-1123.

Hausken K., Moxnes J.F. (2005): “The Dynamics of Crime and Punishment” International Journal of modern Physics 16 (11) pp 1701-1736

Hayek F. A. (1967). "Studies in Philosophy, politics and economics.University of Chicago Press

Hayek F. A. (1973-79). "Law, Legislation and Liberty: A new Statement of the Liberal Principles of justice and Political economy vol.1-3.Routledge

Levitt S,. Miles T. (2006). "Economic contribution to the understanding of crime." Annu.Rev law and social science 2 pp.147-164.

Lucas R. E. (1976) Econometric policy evaluation: A critique.Carnigie-Rochester conference series on public policy vol.1 pp.19-46

.

Lucas R.E. (1990) Why Doesn´t Capital Flow from Rich to Poor Countries? *The American Economic Review*, vol.80, pp.92-96.

Marceau N., Mongrain, S (2011) Competition in law enforcement and capital allocation, *Journal of Urban Economics* 69 (1) pp. 136-147.

Miller, W. I. (2005).*Eye for an Eye*, Cambridge University Press.

Polinsky M., Shavell S. (2006). Public enforcement of law. Handbook of law and economics. Amsterdam, Holland/elsevier sci.

Raine, A. (1993). *The Psycopathology of Crime: Criminal Behavior as a Clinical Disorder*, Academic Press, San Diego California.

Raphael S., Winter-Ebmer R. (2001)."Identifying the Effect of Unemployment on Crime."*Journal of Law & Economics*, 44(1):pp. 259-284.

Sherman L. W. (1992). *Attacting Crime: Police and Crime Control.* Chicago, University of Chicago Press.