Crime and welfare in a face-to-face economy
The Rationality of *lex tallionis*

Edited conference paper, Toronto (CLEA)

By

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"One pound of flesh. No more, no less, than one pound of flesh"

Shylock, Merchant of Venice – Act IV, Scene I

Abstract:

Today, it is common knowledge among many criminologist in the constructivist sense, that the classical principle of an "eye for an eye"(lex tallionis), which throughout history has been an institutional framework in many cultures, is a primitive and irrational idea of punishment. As Gandhi famously put it; "an eye for an eye makes the whole world blind." How could it possibly be rational to punish a bankrupt and poor criminal? (Excluding the immoral utility of vengeance).

This paper shows, that if the probability of punishment is 1 (a face to face economy), a reasonable assumption in the days of Hammurabi, Lex tallionis would actually lead to efficiency as long as the "demand for crime" is a convex function, approximately close to 1/x. Or, alternatively, that the elasticity of punishment and crime, is close to h/x, where h is harm done, and x is number of offenses. We will show, that this result holds regardless of whether criminals are bankrupt or not.

Introduction

In the question of crime and punishment, the discussion really hasn't changed for thousands of years. One point of view suggests that punishment is all about equalizing things between the offender and the victim. This classical view dates back to the sixth king of Babylon, Hammurabi (1792 BC–1750 BC), who wrote down 282 legal codes[1], where the most famous was code 196; "*if a man put out the eye of another man, his eye shall be put out.*" This code of law is also stated as *lex tallionis* (the law of retaliation). Of course, the term *lex talionis* did not always refer to literal eye-for-an-eye codes of justice, but was merely a way of thinking, trying to maintain equilibrium in the social order. Monetary compensation was surely an acceptable way of creating justice, especially for small crimes.

The second point of view thinks differently about crime and punishment. This view is not bounded on the victim's loss, but shift instead focus, to the structure of preferences, among the offenders. Punishment, in this line of thinking, is entirely a pedagogic or psychological instrument, a way of teaching "bad" people, how to behave. Everything of course therefore depend on, which kind of pedagogic school one belongs to. Some people think that criminals should be tortured or humiliated. Other thinks they should be healed by love or forgiveness. As Jesus of Nazareth put it; "*You have heard that it was said, "An eye for an eye and a tooth for a tooth". But I say to you, do not resist an evildoer. If anyone strikes you on the right cheek, turn to him the other also. (*Mt5:38–39, NRSV). In modern times, this line of thinking has been very popular among western intellectuals, and without doubt,has been a great source of skepticism, toward punishment and imprisonment, which many criminologists seems to agree upon, is not a very efficient way of learning how to behave.(Sherman 1992, Bayley 1994, Pinker 2002)

This raises the question: Are the classical normative quest for justice and the weights of lady justice in perfect equilibrium, most at all, built on religious superstition? It could surely be

[1]There were actually 281, because there is no code 13. The number 13 was considered an evil an unlucky number

seen in that way. What´s is the point in torturing a poor bankrupt criminal and throwing him to jail? Does it really make him better? Are we not just creating a huge welfare loss?

This positive argument is, as I will show below, not entirely wrong. Many things depend critically on how elastic the supply of crime is, regarding punishment (the price of doing crime) And nobody of course disagree, that sometimes it seems reasonable to believe that criminal behavior is not very elastic in respect to prices. But one cannot use this as a general rule. If criminal preferences are a general trait in human nature we are not able to conclude anything observing individual criminals. In such a case the target for welfare optimum is the marginal criminal, which is precisely the underlying rationality behind lex tallionis. Actually, the idea of an “eye for an eye” could be seen as a generalized “middle position”, stabilizing the social system, with a very easy to follow/understand rule, at least in a face-to-face economy, where punishment is certain. And we will see, the surprising answers, that the departure from the standard model of externalities simply doesn’t means a lot. At least if the elasticity of crime is close to h/x, which means that the “demand for crime” could be states approximately by 1/x

Assumptions

Every analysis of human action and welfare economics has to deal with an assumption, about the aggregate welfare function. The analysis could of course be obtained for any general claim about the welfare function, but we would simply proceed with the standard assumption of an egalitarian utility function at the aggregate. Our main purpose, as a central planner, is to maximize such a function.

Further we will assume that agents are heterogenic and therefore have different net gains of welfare from doing harm to others. Further we will assume that preferences is complete, transitive, continuous, and strictly monotonic, such that an continuous real-valued function $u: \mathbb{R}^n_+ \to \mathbb{R}$, and represent the binary relation $\succsim$. General speaking this mean that people would act, if their gain outweighs their loss:

$$U_i > 0 \text{ or } G_i - \tau_i > 0$$

Where G is gain from person i´s action, and τ_i is the general loss the agent is facing, if he is doing that action. We would think of τ as punishment (taxes, fines, imprisonment, torture etc.) and we will think of G and τ in monetary terms. The harm each person is doing to others, because $G_i - \tau_i > 0$, is denoted by h. We will imagine that G is distributed by a density function Z(i). This means that we are thinking in terms of heterogeneous agents.

This, of course, could be seen as a little odd, because we normally thinks that people doing harm to others, have much lower benefit, than their victims loses. But if this really was the case, in all circumstances, no crime would occur if τ was higher than the offender's gain. There can be many reasons why the gain from doing crime for the individual could be very high – think of a woman, who desperately wants to feed her baby, or two persons fighting, and therefore in a split moment in time and space, actually prefer punishment rather than move away and stop fighting (Polinsky 2006). The main argument, and I believe important main argument, is therefore, that gains from crime comes first and punishment comes later, which allow us to think of agents i punishment as:

(1) $$\tau_i = \sum_{t=1}^{T} \theta^t \varphi_t \qquad i = (1 \ldots\ldots N)$$

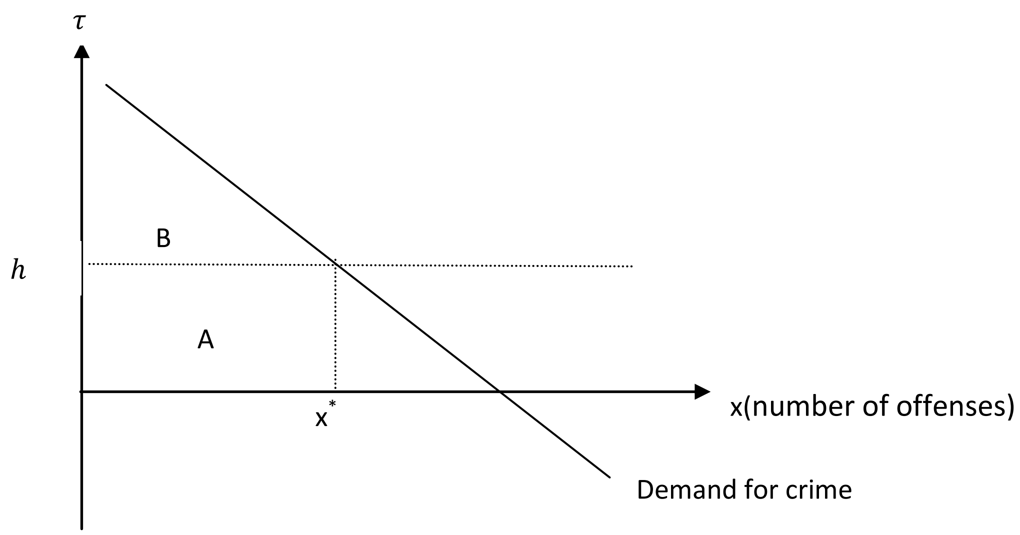
Where θ is the time preference and φ is the stream of punishment for any point in time t. N is the number of agents under consideration. The idea that criminals sometimes could be very impatient (irrational ?!) is of course not new (Gottfredson 1990, Mccrary 2009) and this would lead to a very inelastic demand for crime curve. For now, however, we will not discuss which preferences are "irrational" or "rational" compare to some ethical standard, but rather to maximizing social value, and thereby not to harm offender's more than necessary. As Becker introduced his 1968 article:

> *"The main purpose of this paper is to answer normative questions, namely, how many resources and how much punishment should be used to enforce different kind of legislation? Put equivalently, although more strangely, how many offenses should be permitted and how many offenders should go unpunished? (Becker 1968).*

To answer this question, the paper is organized in the following way. First I will we show, that this proposed set-up, would easily lead to the standard main conclusion from the theory of externalities, namely the result, that people should pay full compensation, from the harm they do to others. So, it is clear that the theory of crime is more or less, a special case of the theory of externalities. Second I will ask; What will happen when the criminals are bankrupt? This seems highly relevant, especially for high-cost crimes. Thirdly we ask, what if it is costly to punish, bankrupt criminals.

The standard case, where Fines is possible

The standard case is rather obvious. If gains are distributed in such a way that they can be represented by an aggregate "demand for crime function", our goal will be to find the optimal level of crime, x*, which of course is equivalent to find the optimal degree of punishment, τ^*. For simplicity we will in the following try to find x*, instead of directly move to τ^* as otherwise is normal procedure. See for example (Polinsky 1991, Garoupa 2000, Polinsky 2006)



(Figure 1: Given a demand for crime curve, and given h, our goal is to find x*, such that we maximize welfare, W)

Proposition 1

Suppose we have the the function $W = \int_0^{x^*} \tau(x)dx - hx^*$. The solution to this problem is:

(2) $$\tau(x^*) = h$$

Hence, if transfer of wealth between evildoer and victims is costless and a monetary compensation vector exists, the optimal monetary fine is "an eye for an eye". In such a case the punishment should fit the crime.

Proof:

The function $W = \int_0^{x^*} \tau(x)dx - hx^*$, can be rewritten as$W = T(x^*) - T(0) - hx^*$. Because we want to maximize welfare we have; $\frac{dw}{dx} = \tau(x^*) - h$. Setting $\frac{dw}{dx} = 0$, and we get: $\tau(x^*) = h$

Example:

Suppose that we have the constant elasticity case, such that$\tau(x) = \left(\frac{x}{a}\right)^{-\frac{1}{e}}$, where e is the elasticity. Because τ(x)=h in optimum, the reciprocal is $x^* = ah^{-e}$

$$W = \int_h^{\infty} (ah^{-e})dh => W = \lim_{\tilde{h}\to\infty} \int_h^{\tilde{h}} (ah^{-e})dh => W = \lim_{\tilde{h}\to\infty} \left[\frac{a}{1-e} h^{1-e}\right]_h^{\tilde{h}} =>$$

$$W = \frac{a}{1-e}\left[\lim_{\tilde{h}\to\infty} h^{1-e} - h^{1-e}\right]$$

This will of course only converges if e>1. For any lower elasticity, wealth would be infinite, and there would be no numerical solution.

As a numerical example, suppose that h=2, a=3 and e=2. Total wealth is:

$$w = \frac{3}{1-2}\left[\lim_{\tilde{h}\to\infty}\frac{1}{\tilde{h}} - 2^{-1}\right] = -3[0-0{,}5] = 1{,}5$$

And the total welfare gain is +1,5. Hence is should be clear that in any case, a society is better off, using the rule $\tau(x^*) = h$.

What if criminals are bankrupt?

The above is of course a standard first-best solution. But it is a very important solution, because it shows the close connection between the standard theories of externalities and theories between crime and economics. Our primary goal is to secure that people initialize the cost for others in their action and it should be of no surprises, that this is done, when people are paying by themselves for the damage done. However, as many in the literature of crime and economics have pointed out(Polinsky 1991, Garoupa 2000), one main problem is, that criminals are not always able to pay full compensation for their actions, and that the rule $\tau(x^*) = h$ therefore is not feasible. More precisely, what we do mean by bankruptcy is:

(3)

$$h \geq a(0) + \int_0^t w(t)e^{-\int_0^t ds}dt$$

Where $a(0)$ *is* the value of individual assets at time zero and w is the accumulated sum of future labor income.

So if harm done is greater than the offender's initial wealth, the offender is not able to compensate society or victims by some monetary transfers. In such a case there is clearly a scope for some insurance companies, which fully or partly would be able to compensate the victims. But we are now left with a general welfare loss, which seem to be larger, as society moves to more and more costly punishment, not only to the offender but also, at least in case of imprisonment cost, also costly to society.

Proposition 2

Suppose that $\mathrm{h} \geq \mathrm{a}(0) + \int_0^t \mathrm{w}(\mathrm{t})\mathrm{e}^{-\int_0^t \mathrm{ds}}\mathrm{dt}$, *that means that the offender is bankrupt. Suppose further that any punishment (creating negative welfare for the offender) could be done* *<u>without further cost for society</u>. In this case a central planner wants to maximize*

(4) $$\mathrm{W} = \int_0^{\mathrm{x}^*} \tau(\mathrm{x})\mathrm{dx} - \tau(\mathrm{x}^*)\mathrm{x}^* - \mathrm{hx}^*$$

And this problem has the solution (maximum or minimum):

(5) $$-\tau(\mathrm{x}^*)\mathrm{n} = \mathrm{h}$$

Where n is the n-power to a function describe by$\tau(x) = x^n \ and \ \tau'(x) = nx^{n-1}$. Hence for functions where n=-1, we are ones again left with the solution $\tau(x^*) = h$, which could be a maximum or a minimum depending on whether $\frac{\mathrm{d}^2\mathrm{w}}{\mathrm{d}^2\mathrm{x}} \gtreqless 0$

Proof

$\mathrm{W} = \int_0^{x^*} \tau(x)dx - \tau(x^*)x^* - hx^*$ Could be rewritten as $\mathrm{W} = T(x^*) - T(0) - \tau(x^*)x^* - hx^*$ [2] Optimum implies that: $\frac{\mathrm{dw}}{\mathrm{dx}} = \tau(x^*) - [\tau'(x^*)x^* + \tau(x^*)] - h = 0$ or $-\tau'(x^*)x^* = h$. For any function $\tau(x) = x^n \ and \ \tau'(x) = nx^{n-1}$, this implies $-\tau(x^*)n = h$. Finding out whether the solution is an optimum requires$\frac{\mathrm{d}^2\mathrm{w}}{\mathrm{d}^2\mathrm{x}} < 0$. We have $\frac{\mathrm{d}^2\mathrm{w}}{\mathrm{d}^2\mathrm{x}} = -[\tau''(x^*)x^* + \tau'(x^*)]$,

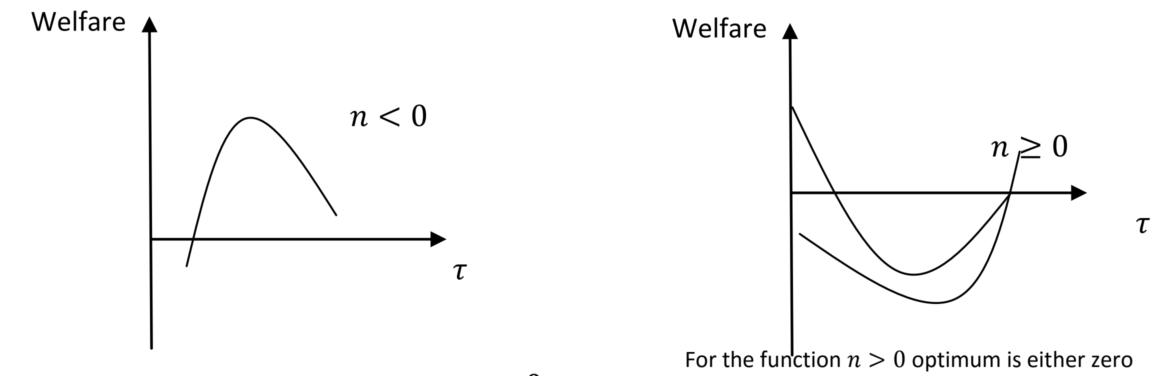
[2]For any function $f(x) = cx^n + k$, therefore could be evaluated as $w = \left[\frac{c}{n+1} - c\right] x^{n+1} - hx$. Hence it is obvious that for any positive value of x to exist, welfare can only be positive if and only, n<-1. If n=-1, welfare is infinite and therefore not defined.

which implied that $\tau''(x^*)x^* > \tau'(x^*)$ or as $\tau'(x^*)n > \tau'(x^*)$. Because $\tau'(x^*)$ is a negative number, any function, where $n \leq 0$, implies an optimum.

Example 2

Suppose we have the "demand for crime" function as $f(x) = 1/\sqrt{x}$. Wealth is given by: $w = \int_0^{x^*} \frac{1}{\sqrt{x}} dx - \frac{1}{\sqrt{x}} x - hx$or $\lim_{\tilde{x}\to 0} \int_{0+\tilde{x}}^{x^*} [2x^{0,5}]_{\tilde{x}}^{x^*} - \sqrt{x^*} - hx$. Because this integral is defined we get (ignoring *): $w = 2x^{0,5} - x^{0,5} - hx$or $w = \sqrt{x} - hx$. Finding wealth optimum means that;

$\frac{dw}{dx} = 0,5x^{-0,5} - h = 0 => x = 1/4h^2$. Numerically, suppose that h=0,2 X in optimum is 6,25 and welfare is positive 1,25. The price of crime is in this example 0, 4, and the optimal solution therefore implies over deterrence.



For the function $n \leq 0$ an optimum exist when $\tau > 0$

For the function $n > 0$ optimum is either zero punishment or $\tau \rightarrow max$

(Figure 2: Depending on how convex the demand function really is, there could be underdeterrence or overdeterrence)

This makes of course a lot of sense. In the case of bankrupt agent, there are three possibilities, depending on how much gain the criminals get, compare to the victims loss. In the case, where a lot of harm is done and the gain for the offender is rather small, high punishment would simply deterrent all criminals and we are left with W=0. In cases with rather small loses for the victims, the optimal solution could be simply not to do anything. In cases where

$n \le 0$, and the demand for crime is very convex, the optimal solution is underdeterrence but positive punishment.

It is interesting, that for the function $\tau(x^*) = x^{-1}$, (n=-1) our optimum collapse to:

(6) $$\tau(x^*) = h$$

Once again the best outcome is "an eye an eye". However in such a case welfare is not defined because $T(0)$ is infinite (remember that $\int \frac{1}{x} = lnx$). This understated, however, only understated the fact that $\tau(x^*) = h$ is an optimal solution. Hence, for any demand for crime function, very close to 1/x, "an eye for an eye" is an optimal solution.

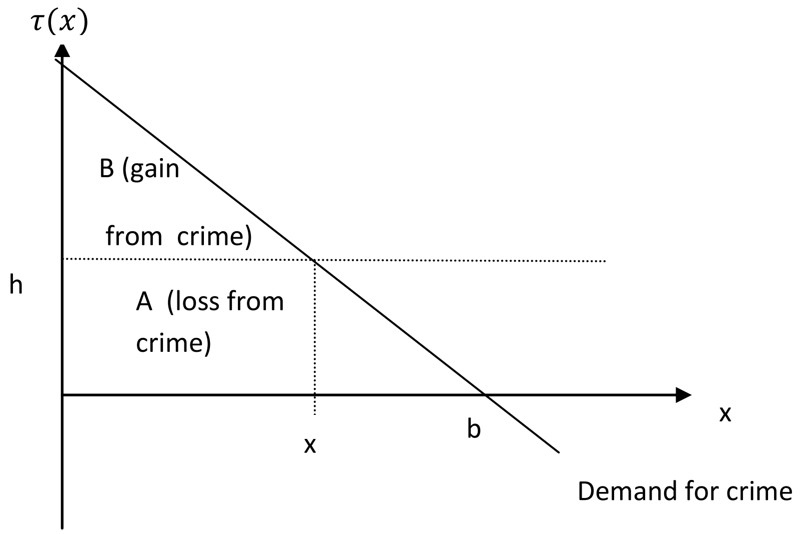


Figure 3 (For any linear function and where agents are bankrupt, there is no "middle way" optimum x*. Either we move to maximum punishment which implies w=0 or we move to zero punishment . In such a case $w = \int_0^b \tau(x) - bh > \int_0^{x^*} \tau(x)dx - \tau(x^*)x^* - hx^*$ for any x* . This make sense. Sometimes the harm done is always smaller than the benefit for the evildoer, hence we move to no punishment. Or otherwise the harm done is always higher than the benefit for the evildoer, and we therefore move to maximum punishment and end with zero crime.

The use of costly punishment

When it comes to punishing bankrupt criminals, throughout history. creativity and fantasy, have been great (Miller 2005). Nowadays, it seems that we all end up with the same kind of punishment, namely prison. The reason is undoubtful a humanitarian one. The price of doing torture or cut of limbs is theoretical close to zero for society. Using prison, and therefore letting people pay for their action with their freedom (time cost or alternative cost), is of course costly for society. But we cannot underestimate the psychological cost and therefore the welfare loss from torture for society. So torture implicit means a welfare loss. Most people simply hate to see other people suffer (ruling out the benefit from revenge), even though that, as we have seen, that suffering (or a price of doing something bad) can be viewed as necessary at the aggregate level. Costly imprisonment can therefore be the best alternative.

Proposition 3

In case of costly punishment, our goal is now to maximize the function[3]:

(7)

$$W = \int_0^{x^*} \tau(x)dx - \tau(x^*)x^* - hx^* - c\,\tau(x^*)x^*$$

Where c is the extra cost of "creating" a welfare loss to the offender. Hence c is a parameter > 0. Note that if $\tau(x^*) \to 0\ or\ x^* \to 0, then\ c\tau(x^*)x^* \to 0$. This is of no surprise. If we choose not to punish any criminals, maybe because we think that the harm done is very low, the cost

[3]Polansky and Shavell shows the following: *suppose we want to maximize* $\int_{d(s)}^{\infty}(g - h - d(s) - cs)z(g)dg$,

g=gain an individual obtains if he commits the harmful act; z(g) = density of gains among the individuals; h= harm (or estimated harm) causes by an individual if he commits the harmful act s = sentences, jail, torture or body parts, d(s) = disutility from sentences, d(0)=0, and d´(s)>0 c=cost of implement pr. sentences. Solving gives; $\int_{d(s)}^{\infty}(d'(s) + c)z(g)dg=(h+s)(dz(d(s))/d(s)$, which could be stated as Marginal cost and marginal gains. This is of course equal to the above, but could be considered to be a more general statement.

of harming criminals would be zero. If we choose the price of doing crime arbitrary high, there would again be no loss, simply because nobody wants to commit crime.

Let $S = 1 + c$, we then rewrite the above to:

(8) $$\mathrm{W} = \int_0^{\mathrm{x}^*} \tau(\mathrm{x})\mathrm{dx} - \mathrm{hx}^* - \mathrm{s}\,\tau(\mathrm{x}^*)\mathrm{x}^*$$

With the solution:

(9) $$\tau(\mathrm{x}^*) - \lceil \mathrm{s}\tau(\mathrm{x}^*)\mathrm{n} + \mathrm{s}\tau(\mathrm{x}^*)\rceil = \mathrm{h}$$

Proof

If $\mathrm{W} = \int_0^{x^*} \tau(x)dx - hx^* - s\,\tau(x^*)x^*$ an optimum implies that $\frac{\mathrm{dw}}{\mathrm{dx}} = \tau(x^*) - \lceil s\tau'(x^*)x^* + s\tau(x^*)\rceil - h$=0. Using the power rule as in proposition 2, this could be stated as $\frac{\mathrm{dw}}{\mathrm{dx}} = \tau(x^*) - \lceil s\tau(x^*)n + s\tau(x^*)\rceil - h$ =0 or $\tau(x^*) = h + \lceil s\tau(x^*)n + s\tau(x^*)\rceil$

Depending on the cost, the weight in the equation for the term $\tau(x^*)x^*$ is now greater, because we assume it costly to produce a welfare loss to the offender. So in cases of crime, we are "now all losers". Note that if s=1 or c=0, we are of course left with proposition 2.

Because $\tau'(x^*) < 0$ and $s, x^*, \tau(x^*) > 0$, we simply are not able to tell wherever $\tau(x^*) < h$ or if $\tau(x^*) > h$, that is we are not able to tell, whether (4) would end up in under deterrence or over deterrence. This all depend on how convex the function really are. There is, however, an interesting spot, where $\tau(x^*) = h$ and the rule, "eye for an eye" does apply, which lead to:

Proposition 4

Even if producing of punishment to the offender is costly to society, an "eye for an eye" solution $\tau(x^*) = h$ exist. This will be the case for the "demand for crime" function moves close to 1/x, and the cases, where $\tau(x^*) \to 0$ (non serious crimes) or $s \to 0$ (fines is possible)

Proof:

We have that if $[s\tau(x^*)n + s\tau(x^*)] \rightarrow 0$, then $\tau(x^*) \rightarrow h$. Therefore it must be true that if $n \rightarrow -1,\ \tau(x^*) \rightarrow 0$ or $s \rightarrow 0$ the optimal solution for w^* -> $f(x^*) = h$

We therefore have the following possibilities when agents are bankrupt:

n	Result	commentary
$n < -1$	$\tau(x^*) < h$	Some underdeterrence could be viewed as optimal
$n > -1$	$\tau(x^*) > h$	Some overdeterrence is optimal (see example 2, where n=-0,5)
$n = -1$	$\tau(x^*) = h$	“eye for an eye”

This means that of n=-1, meaning that the “demand for crime” around the observed level of crime, is close to 1/x, which generate a elasticity of h/x . In such a case, an "eye for an eye" would be the optimal strategy.

Conclusion

We have from the above shown, that from a very general and standard economic starting point, the best solution, regarding crime, is, “an eye for an eye”, meaning that the offender must pay compensation for the victim’s loss. In such a case, all prices and externalities are included in people’s preferences. When people are bankrupt and compensation is no longer possible, it seem pointless and meaningless to punish the offender – it all just cost. But the effect from punishment in such a case critically depends in the effect on others. How high is the elasticity of crime regarding prices? Welfare for society is of course, in the bankruptcy set-up, lower, but we have shown that we cannot rule “eye for an eye” out as an optimal strategy. The reason is that punishment really function as the same way as fines, thereby securing that people are initializing cost into their actions. We have seen that if “the demand for crime” is very convex, which could be stated as “the criminals have extremely large gains from doing crime (high time preference), or maybe as non-economist would put it, they are “irrational”, punishment would lose its power and some underdeterrece is optimal, because there is really no point in punishing more than necessary to the offender. (Excluding utility of vengeance). If criminals don’t have very large benefit from crime, they are “very rational”; punishment could be a very effective way of combating crime. If one believes that the demand for crime is not perfectly observable, but somewhat close to n=1 “an eye for an eye” strategy could be seen as a very easy rule to follow and understand. Hence, “an eye for an eye” is more than just an irrational psychological trick, rooted in a wish of vengeance.

However, the simplicity would of course break down in a more modern and complicated economy, where enforcement is not certain. In such case lex tallionis could be stated as (Benthams first law):

(10)
$$\tau(x^*) = \frac{h}{\rho}$$

Where ρ is the probability of detection and conviction$\rho \in [0,1]$.If $\rho \to 1$, we have of course the classical principle.

It seems important to understand that as$\rho \to 0$, the divergens between τ and h becomes greater and greater. Hence, the basic behind the “eye for an eye” principle, breaks down, because it would lead to substantial underdeterrence. This, I believe, explain, why modern societies and large social systems, as industrialization and urbanization moved forward, was forces to leave the easy and understandable principle of lex tallionis. Society’s didn´t leave because lex tallionis was irrational, but because it wasn’t enough.

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